$$=\tfrac{1}{2}(2-e^2)\int_0^{\frac{1}{2}\pi} dx\sqrt{1-\frac{e^4}{(2-e^2)^2}\sin^2\lambda}=\tfrac{1}{2}(2-e^2)E\left(\frac{e^2}{2-e^2},\ \frac{\pi}{2}\right)=\tfrac{1}{2},\text{ if } e=1.$$

Also solved by *OTTO GEOKELER, WILLIAM HOOVER, ARTEMAS MARTIN, F. P. MATZ, and G. B. M. ZERR.*

21. Proposed by T. JOHN COLE, Columbus, Ohio.

In the equilateral triangle ABC, AB the base is 10 feet. With B as a center an arc is drawn from C to A; likewise with A as a center an arc is drawn from C to B. What is the volume of the solid generated by revolving the figure about the altitude of the triangle as an axis.

I. Solution by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

The triangle revolving describes a cone the volume of which is $\frac{1}{3}\pi.25.5\surd3=226.725$.

The area of either segment is $50\left(\frac{\pi}{3}-\frac{\surd3}{2}\right)=9.05861$; the distance of the center of gravity of the segment from the center of the circle $=\frac{1000}{12\text{ Area}}$, and from the axis of revolution is $\left(\frac{1000}{12\text{ Area}}-5\sec30^\circ\right)\cos30^\circ=2.9669$.

The product of the area by the path of the center of gravity $9.05861\times 2.\pi.2.9569=168.865$.

Adding to this the volume of the cone, the answer is 395.59 cubic feet.

II. Solution by JOHN DOLMAN, Jr., Counsellor at Law, Philadelphia Pennsylvania.

Let the origin be at the centre of the base. Put the side of the triangle$=2r$, and let $y=$the radius of any circular horizontal section at any variable height x.

Then $V=\int_0^{r\surd3}\pi y^2dx\ldots.(1)$. The equation of the arc forming the side is $4r^2=x^2+(y+r)^2$ from which $y^2=5r^2-x^2-2r\surd(4r^2-x^2)$ which value of y^2 substituted in (1) gives $V=\pi\int_0^{r\surd3}[(5r^2-x^2-2r\surd(4r^2-x^2)]dx$

$$=\pi\left[5r^2x-\tfrac{1}{3}x^3-r[x\surd(4r^2-x^2)+4r^2\sin^{-1}\frac{x}{2r}\right]_0^{r\surd3}$$

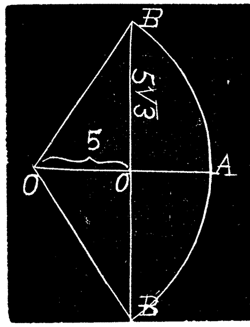
$=\pi(3r^3\surd3-\frac{4}{3}\ \pi r^3)=\frac{\pi r^3}{3}(9\surd3-4\pi)=3.164722r^3$, and putting $r=5$ this gives $V=395.59+$cubic feet.

III. Solution by OTTO GEOKELER, Bloomington, Indiana.

The equation of circle with O as origin is $y^2+(x-5)^2=100$, or $x=5$

$-\sqrt{(100-y^2)}$ on part of curve in question. Then from conditions of problem

$$V=2\pi\int_0^{5\sqrt{3}}\int_0^{5-\sqrt{(100-y^2)}} x\,dx\,dy=\pi\int_0^{5\sqrt{3}}[125-y^2-10\sqrt{(a^2-y^2)}]dy.$$

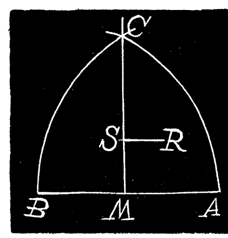


$$=\pi\left[125\times5\sqrt{3}-125\sqrt{3}-1000\int_0^{\theta=\sin^{-1}\sqrt{\frac{3}{4}}}\cos^2\theta d\theta\right]$$

$$=\pi\left[125\times4\times\sqrt{3}-1000\left(\frac{\sqrt{3}}{8}+\frac{\pi}{6}\right)\right]=\pi\left[375\sqrt{3}-\frac{1000}{6}\pi\right]=\frac{125\pi}{3}(9\sqrt{3}-4\pi)=395.59+\text{cubic feet.}$$

IV. **Solution by W. WIGGINS, Richmond, Indiana.**

The volume generated is equal to that of cylinder of base CMA, and altitude equal to the length of the path scolpt out by R, the center of gravity of the area CMA. (Guldins theorem)



Taking B as the origin of co-ordinates and BA as the axis of x, we have for the area of AMC,

$$\text{Area}=\tfrac{1}{2}.100.\frac{\pi}{3}-\tfrac{1}{2}.100.\tfrac{1}{2}.\tfrac{1}{2}\sqrt{3}=\tfrac{1}{6}(100\pi-75\sqrt{3}).$$

If x', y' denote the co-ordinates of c, g, of the area CMA, x'. area $CMA=\int_0^{10}x\sqrt{(100-x^2)}dx.$

x'. area$=\frac{1}{3}(100-x^2)^{\frac{3}{2}}$ x'. area$=\frac{1}{3}y^3$, where $y=5\sqrt{3}$.

$\therefore\ x'=\dfrac{[5\sqrt{3}]^3}{3\times\text{Area}}$, or $x'=\dfrac{125\sqrt{3}}{\text{Area}}$, $\therefore\ SR=\dfrac{125\sqrt{3}}{\text{Area}}-5$. Therefore if h denote the altitude of the equivalent cylinder, we have $h=2\pi\left(\dfrac{125\sqrt{3}}{\text{Area}}-5\right)$.

$$V=2\pi\left(\frac{125\sqrt{3}}{\text{Area}}-5\right)\times\text{Area}=2\pi(125\sqrt{3}-5\text{ Area})=2\pi[125\sqrt{3}-\tfrac{5}{6}(100\pi-75\sqrt{3})]=\frac{125\pi}{3}(9\sqrt{3}-4\pi)=395.59+\text{ cubic feet.}$$

Also variously solved by *C. W. M. BLACK, A. L. FOOTE, J. F. W. SCHEFFER, G. B. M. ZERR, J. H. DRUMMOND, F. P. MATZ, and P. H. PHILBRICK.*

PROBLEMS.

28. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

How far from the stage must Miss Love sit in order that she *may see to best advantage* Mr. Rich deliver the valedictory oration ?

29. **Proposed by CHARLES E. MYERS, Canton, Ohio.**

A hen running at the rate of $n=2$ feet per second, on the circumference of a circle, radius $r=50$ feet, is observed by a hawk $a=600$ feet directly above the center,